

Fluency: More than Facts!

Jennifer Hagman and Kelli Wasserman



“To the person without number sense, arithmetic is a bewildering territory in which any deviation from the known path may rapidly lead to being totally lost.”

Dowker (1992)

Session Description

Do your students have fluency?

What does fluency look like in the classroom?

We will investigate these questions through activities, video and discussion. This session will include activities you can implement immediately in your classroom.

Who are we?

Middle/High school
Teacher

Mom of a 3rd and 4th
Grader

Masters in Teaching
Mathematics

Was invited to participate
In grant-funded PD.

Passionate about parent
education.

Executive Board Members of
RSBCMTA

Members of CMC

Members of NCTM

Teach MCPT courses

Lesson Study Facilitators

Professional Development Facilitators

We are passionate about math and
teaching math.

We are lifelong learners.

Taught at CSUSB.

Ignited my interest in
teaching math, and
PD for (math) teachers.

Worked with CMAT, a
non-profit math publisher
of middle school
curricula.

Dedicated to learning
and sharing about
mathematical modeling.

Brain Drain

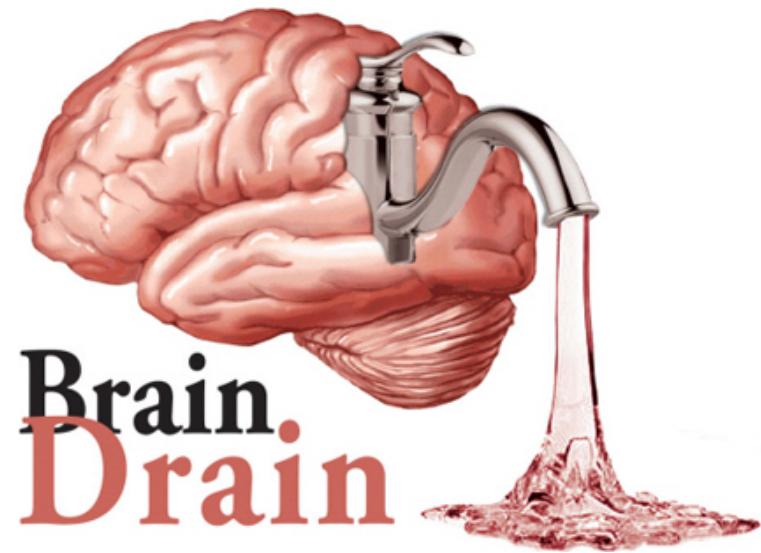
What does each of the following terms mean? How are they similar or different?

Fluency

Number Sense

How do we *develop* fluency?

How do we know when students *have it*?



With a partner, you
will have one minute to
memorize the symbols on
the flash cards.



Can you figure out what this says?



Debrief

How did participating in this activity make you feel?

Were you successful with this task? (Show of hands.)

Memorization only works for about 10-20% of learners.

“Can’t we just teach the way we have always taught to the ones who get it, and go back and teach the “other stuff” (concepts) to the ones who don’t get it?”

DOES THIS HELP?

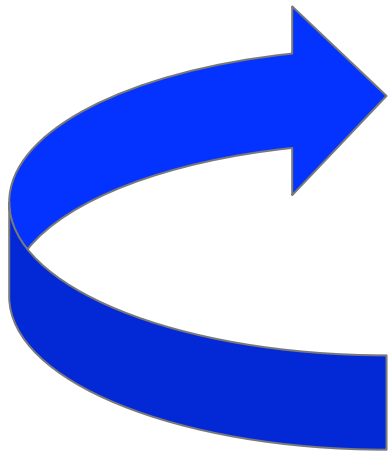
A	B	C
D	E	F
G	H	I

What does this say?

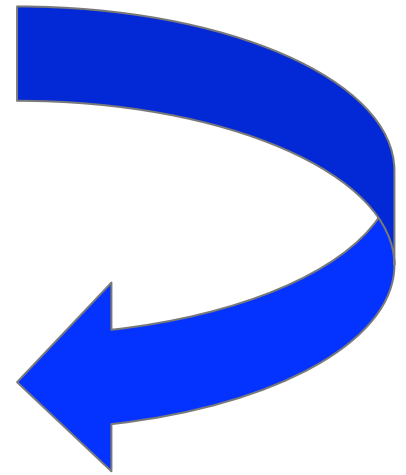


What does fluency look/sound like?

Grace (11)



Fluency



Number Sense

What is Number Sense?

- Number sense essentially refers to a student's "fluidity and flexibility with numbers," (Gersten & Chard, 2001).
- He/She has sense of what numbers mean, understands their relationship to one another, is able to perform mental math, understands symbolic representations, and can use those numbers in real world situations.
- In her book, *About Teaching Mathematics*, Marilyn Burns describes students with a strong number sense in the following way: "[They] can think and reason flexibly with numbers, use numbers to solve problems, spot unreasonable answers, understand how numbers can be taken apart and put together in different ways, see connections among operations, figure mentally, and make reasonable estimates."

What is Number Sense?

Number Sense is... “a person’s general understanding of numbers and operations. It includes using this understanding in *flexible* ways to make mathematical judgments, and to develop useful strategies for handling numbers and operations...”

- *Reys and Yang (1998)*



Explorations with the Hundreds Chart

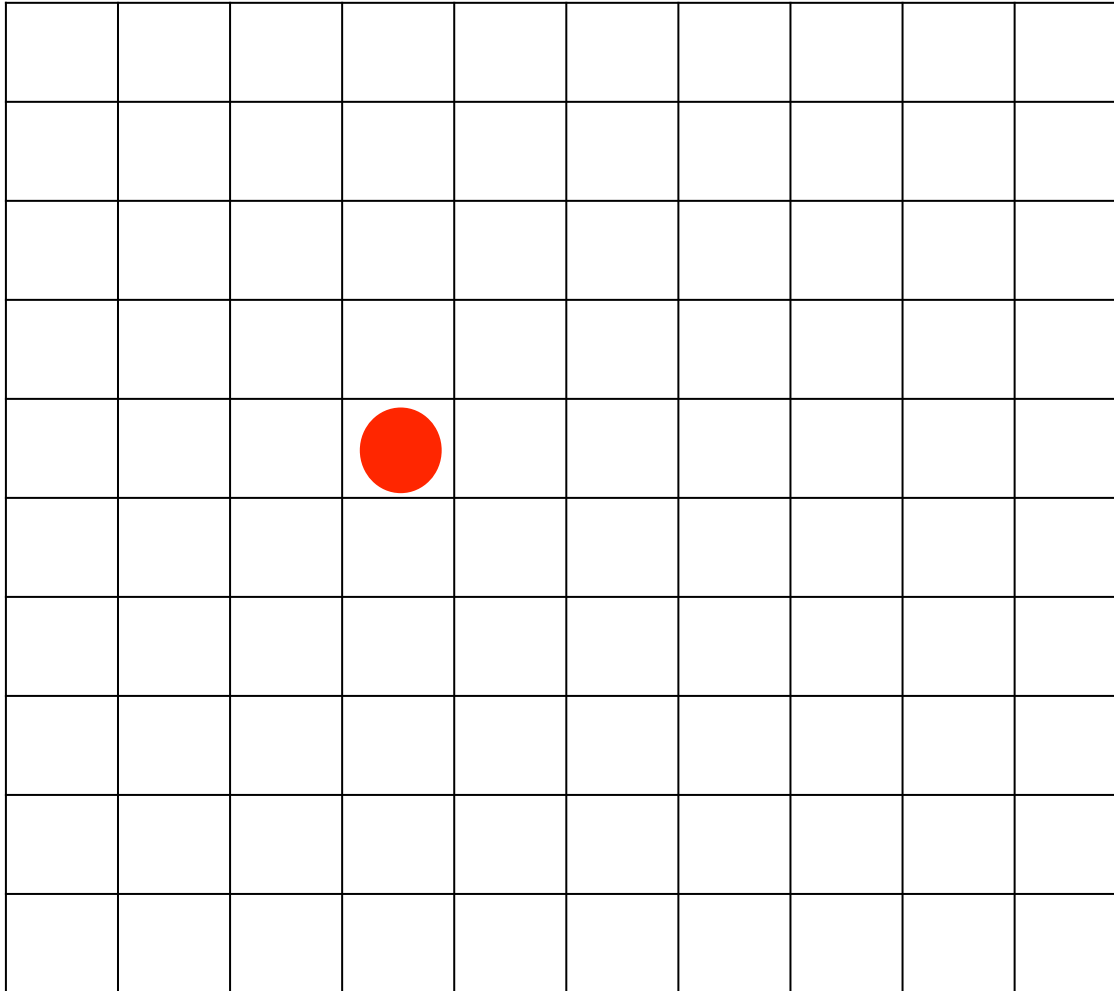
What patterns do
you see?

How are the rows
related?

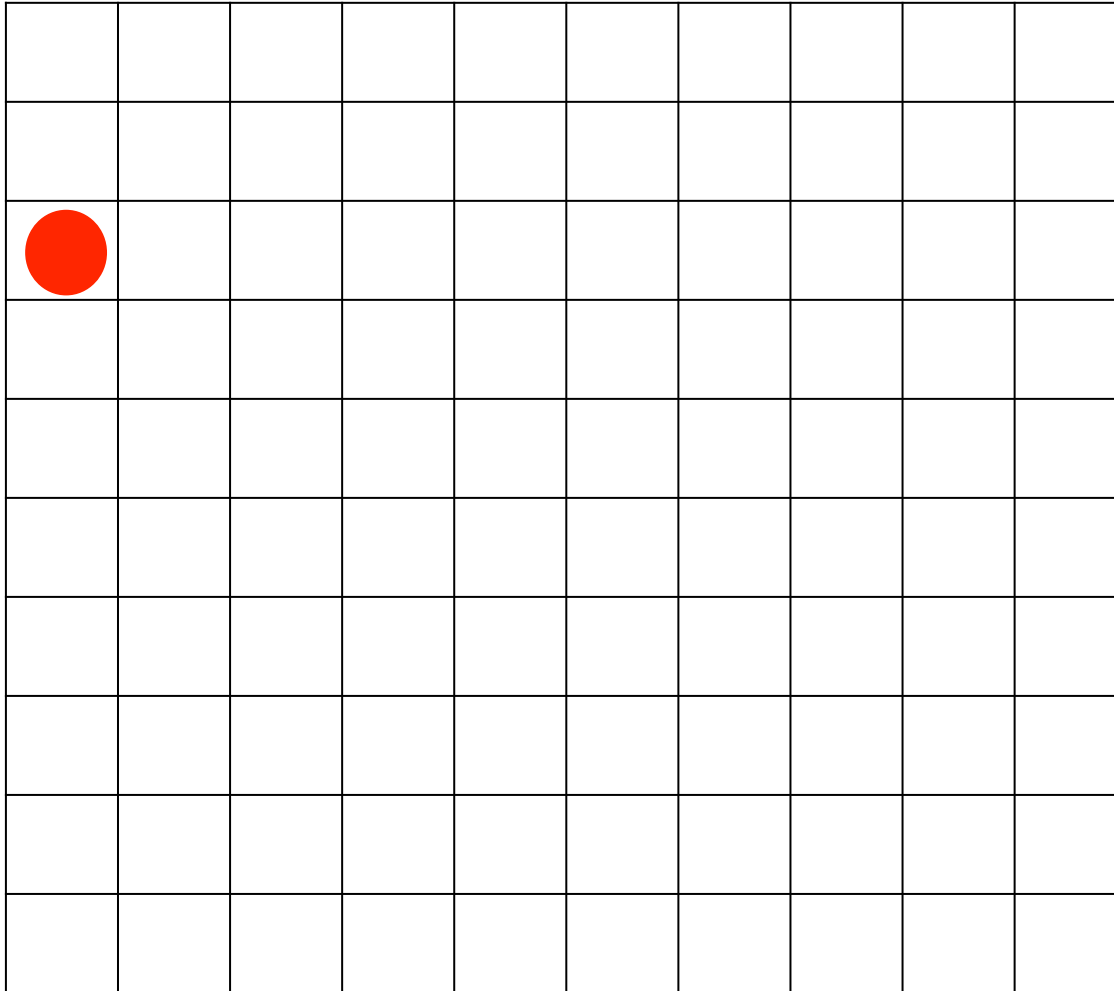
How are the
columns related?

How are the values
on the diagonals
related?

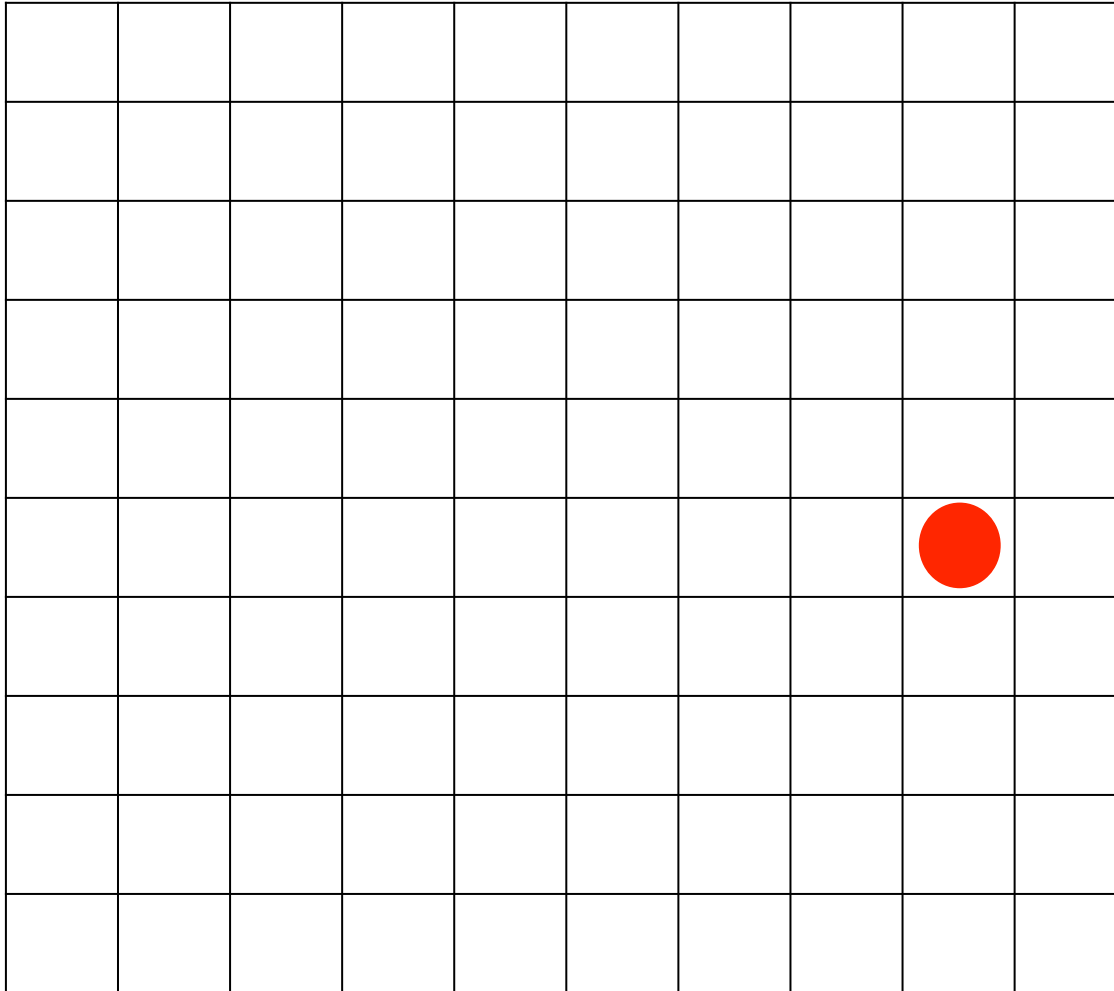
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100



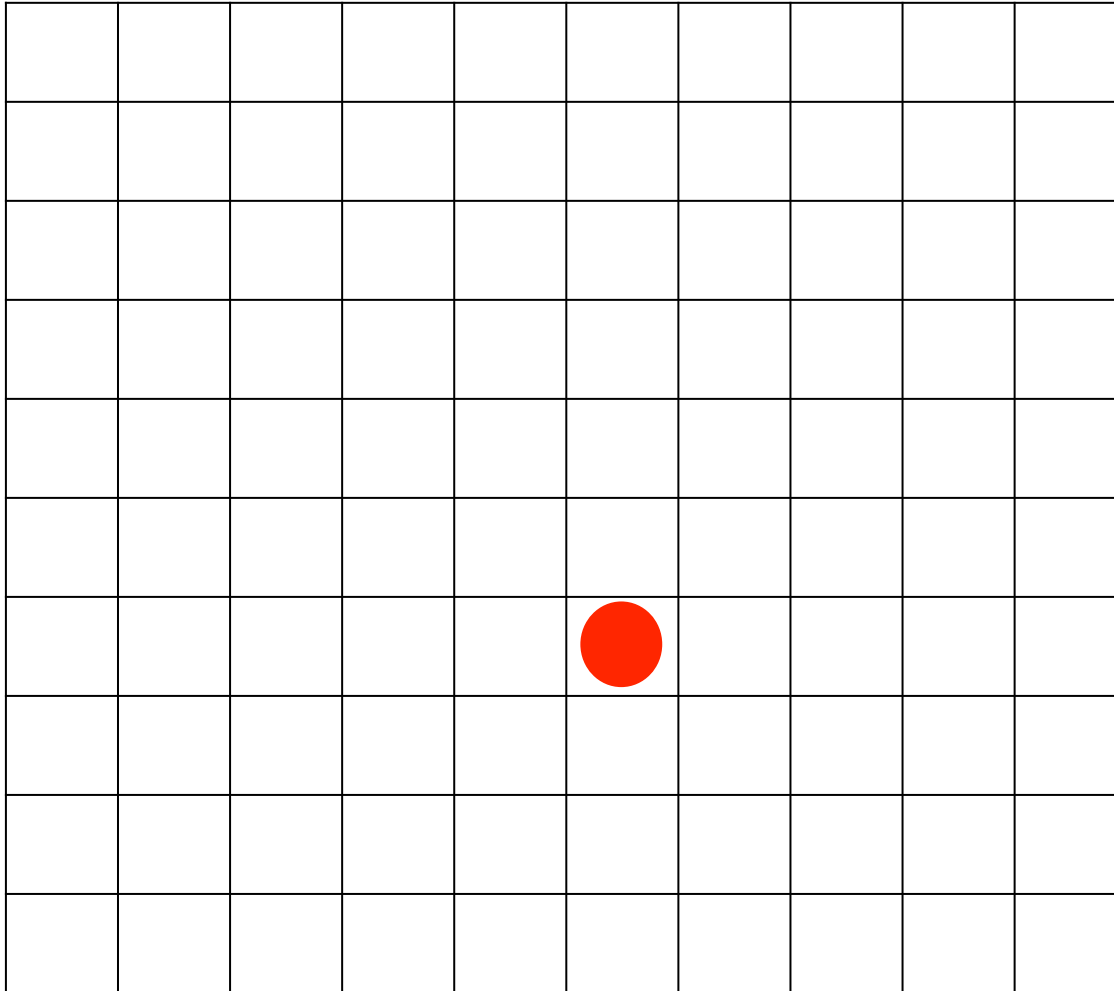
Find the
value of
the
marker.



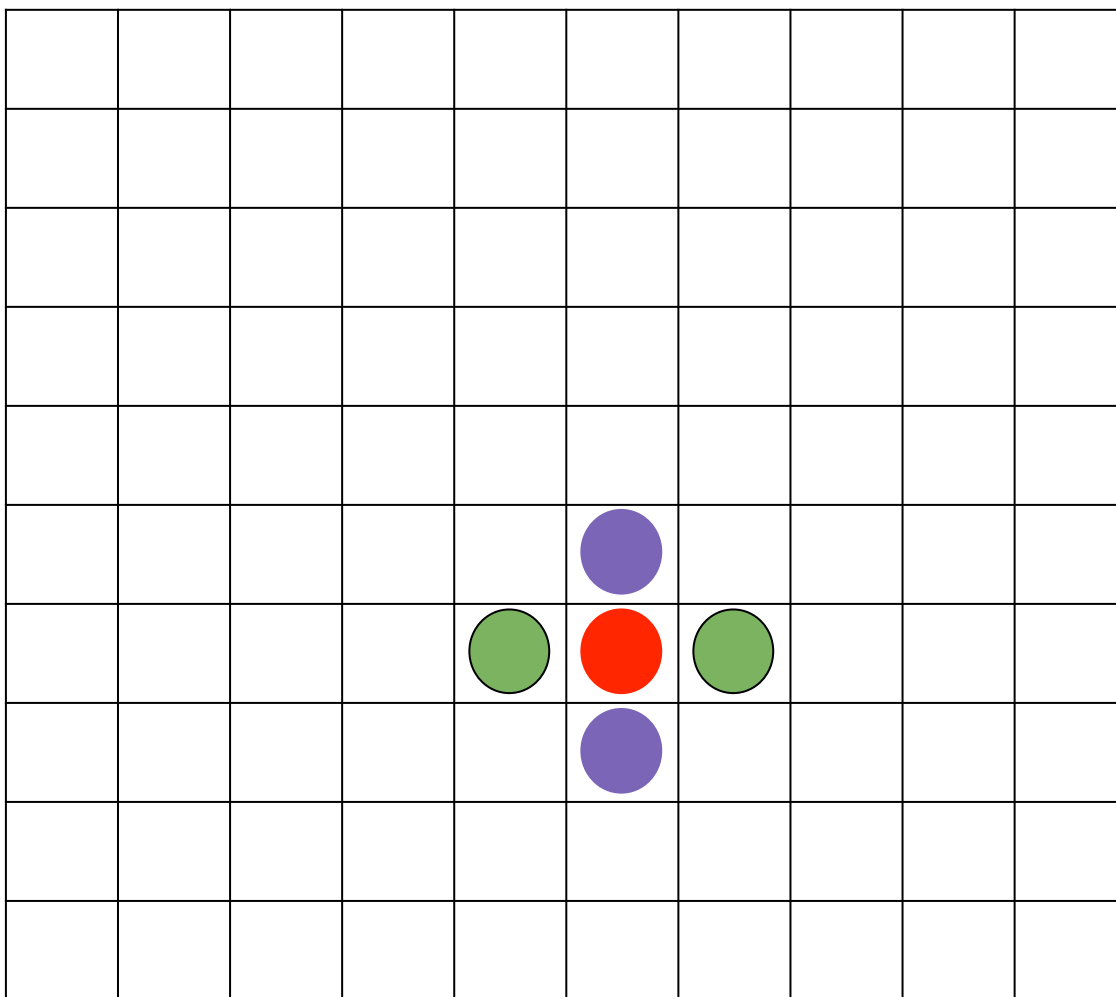
Find the
value of
the
marker.



Find the
value of
the
marker.



Find the
value of
the
marker.

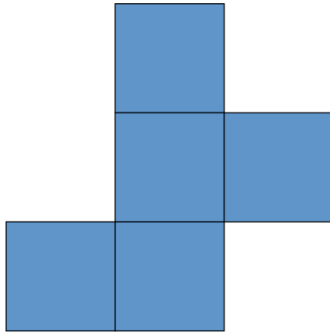


Find the
values of the
purple
markers.

Find the
values of the
green
markers.

Puzzle Pieces

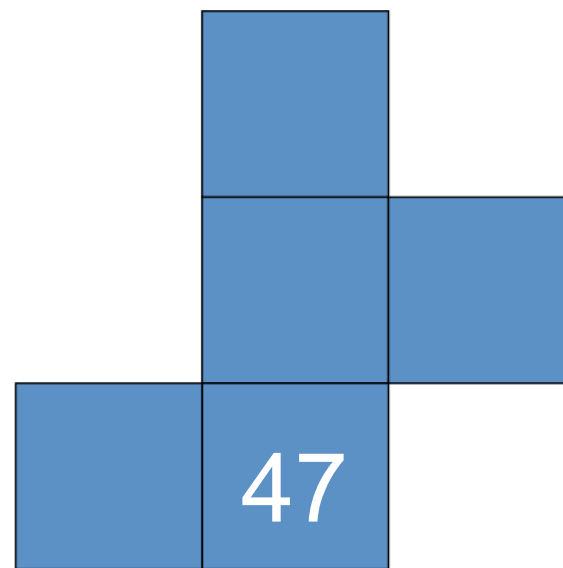
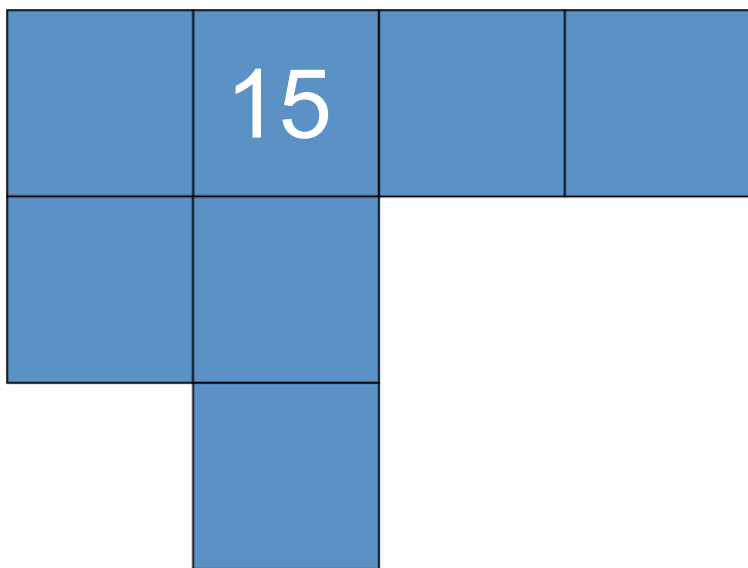
How can you
determine the
numbers that belong
on the puzzle piece?



By Shirley Roath - RCOE

1	2	3	4	5	6	7	8	9	10
11	12	13		15	16	17	18	19	20
21	22	23			26	27	28	29	30
31	32			35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

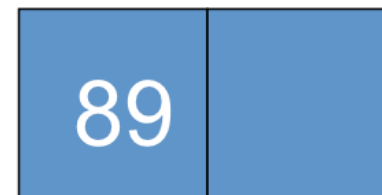
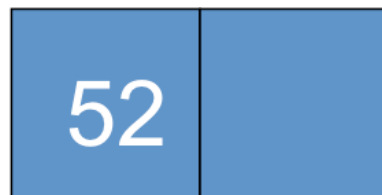
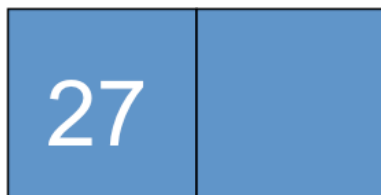
Fill in the missing numbers on these puzzle pieces.



By Shirley Roath - RCOE

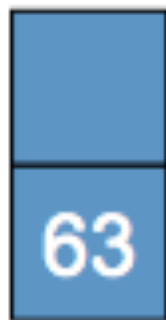
What numbers are missing on these puzzle pieces?

Describe how to find the missing numbers on all of these puzzle pieces.



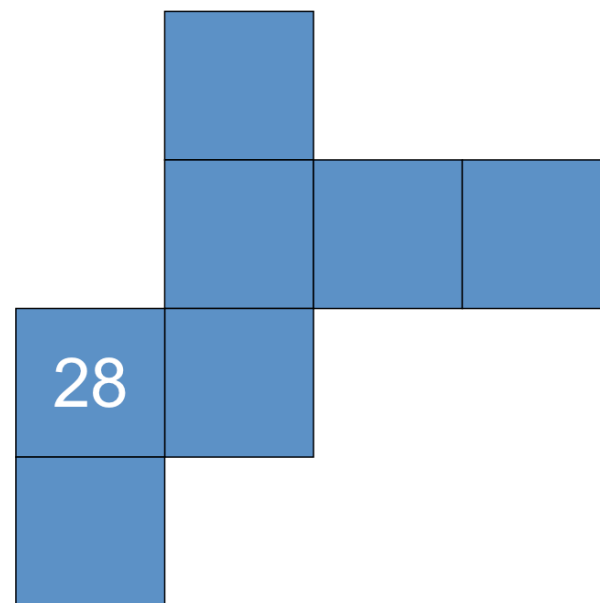
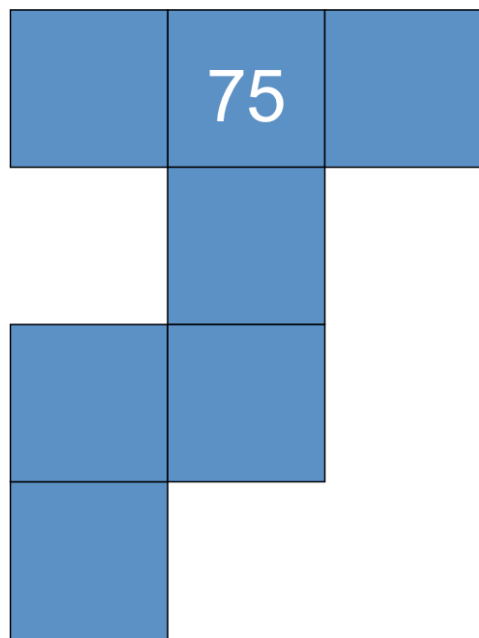
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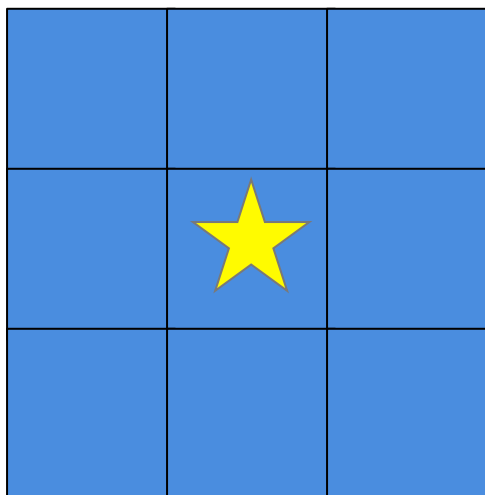
By Shirley Roath - RCOE

Why don't these shapes fit on the hundreds chart?



By Shirley Roath - RCOE

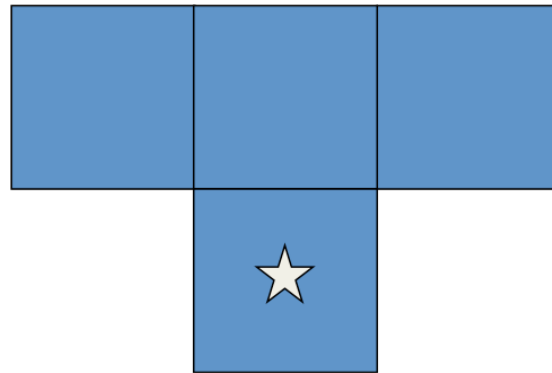
What is the relationship between the square with the star and each of the other squares that surround it?



By Shirley Roath - RCOE

Find all the numbers ...

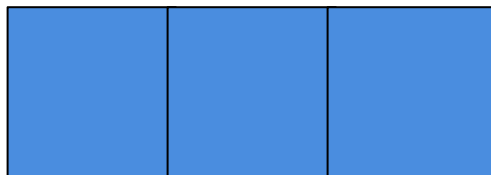
...from 1 to 100 that can replace the star so that the puzzle piece does not fit on the hundreds chart.



By Shirley Roath - RCOE

Find all the numbers...

This puzzle piece fits in a row of the hundreds chart. Fill in the missing numbers so that their sum is 54.



By Shirley Roath - RCOE

Debrief

What was the purpose of this hundred chart exploration?

How did this activity help you describe relationships between numbers?

Would this activity help your students to deepen their number sense?



What is fluency?

We will define fluency as the ability to work with numbers
FLEXIBLY, EFFICIENTLY and ACCURATELY.



Efficiency...

...implies that the student does not get bogged down in too many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of subproblems and making use of intermediate results to solve the problem.

Accuracy...

...depends on several aspects of the problem-solving process, among them careful recording, knowledge of number facts and other important number relationships, and double-checking results.

Flexibility...

...requires the knowledge of more than one approach to solving a particular kind of problem, such as two-digit multiplication. Students need to be flexible in order to choose an appropriate strategy for the problem at hand, and also to use one method to solve a problem and another method to double-check the results.

Mental Math

Solve the following problems using ***mental math*** (no pencil, paper, calculator, or neighbors!)

When you have an answer, indicate with a thumbs up in front of your chest.

If you finish quickly, try to find another way to solve. You may indicate this with additional fingers in front of your chest.

Mental Math

$$28 + 35$$

Indicate you have at least one solution with a 

Strategies for Addition

- Round and adjust
- Compensation (take and give)
- Add up from one number
- Add from the left by place value
- Counting on the hundreds chart

The Basics of Number Talks

Whole class activity centered around *mental math*.

Teacher visually presents *purposefully chosen* problem(s), which may or may not be connected to the day's lesson.

Students explain and justify multiple solution strategies.

Teacher acts as *facilitator* and *recorder*.

Short, ongoing daily routine.

Time required: 5-10 minutes.

Why are Number Talks important?

- ★ A way to dig deep into why mathematical procedures work
- ★ Help students **develop flexibility and confidence** working with numbers
- ★ Help students **build a solid foundation** for future mathematics learning
- ★ Help students **understand** important mathematical ideas
- ★ Help students **make sense** of their own mathematical ideas
- ★ Promote a **spirit of inquiry** and a thirst for knowledge

Mental Math

925-376

Indicate you have at least one solution with a 

Number Talk: Some Strategies

Standard Algorithm

Counting Up

Decomposition

Round and adjust

Use a related fact

Constant Difference

Break apart by place value and subtract left to right

Luke's (8) Solution Method

- Compare Luke's thinking to Grace's thinking.
- What do you notice?
- What do you wonder?

Luke's solution method...

$$\begin{array}{r} 900 + 20 + 5 \\ - 300 - 70 - 6 \\ \hline 600 - 50 - 1 \end{array}$$

$$\begin{array}{r} 4) 200 + 60 + 7 \\ - 70 + 8 \\ \hline 200 - 10 - 1 = 189 \end{array}$$

$$\begin{array}{r} 5) 200 + 20 + 7 \\ - 200 - 70 - 9 \\ \hline 300 - 50 - 2 = 248 \end{array}$$

$\begin{array}{r} 63 \\ 91 \\ + 78 \\ \hline \end{array}$	$\begin{array}{r} 98 \\ 71 \\ + 63 \\ \hline \end{array}$
$220 + 12 = 232$	232
<p>These have the same tens and the same ones, so they have the same answer.</p>	
<p>Math Aids.Com Addition Worksheets</p>	

Adding with Some Regrouping (A)

Find each sum.

9 +9 <u>18</u> 4 +1 <u>5</u> 3 +8 <u>11</u> 1 +6 <u>7</u> 2 +7 <u>9</u> 7 +6 <u>13</u> 4 +4 <u>8</u> 9 +8 <u>17</u> 9 +6 <u>15</u> 2 +9 <u>11</u>	9 +2 <u>11</u> 2 +9 <u>11</u> 8 +6 <u>14</u> 6 +4 <u>10</u> 4 +3 <u>7</u> 3 +5 <u>8</u> 9 +8 <u>17</u> 8 +9 <u>17</u> 9 +5 <u>14</u> 3 +7 <u>11</u>	8 +1 <u>9</u> 6 +6 <u>12</u> 9 +6 <u>15</u> 4 +3 <u>7</u> 2 +8 <u>10</u> 5 +9 <u>14</u> 5 +4 <u>9</u> 2 +3 <u>5</u> 8 +9 <u>17</u> 1 +1 <u>2</u> 4 +4 <u>8</u> 5 +9 <u>14</u> 5 +4 <u>9</u>	5 +4 <u>9</u> 2 +3 <u>5</u> 9 +7 <u>16</u> 6 +9 <u>15</u> 5 +7 <u>12</u> 7 +8 <u>14</u> 5 +9 <u>14</u> 3 +2 <u>5</u> 5 +6 <u>11</u> 6 +1 <u>7</u> 7 +2 <u>9</u> 6 +5 <u>11</u> 3 +4 <u>7</u>	1 +7 <u>8</u> 8 +3 <u>11</u> 4 +8 <u>12</u> 1 +4 <u>5</u> 5 +4 <u>9</u> 6 +7 <u>13</u> 7 +6 <u>13</u> 3 +5 <u>8</u> 6 +1 <u>7</u> 7 +2 <u>9</u> 6 +5 <u>11</u> 3 +4 <u>7</u>	7 +2 <u>9</u> 3 +9 <u>12</u> 2 +6 <u>8</u> 4 +8 <u>12</u> 8 +4 <u>12</u> 9 +2 <u>11</u> 5 +1 <u>6</u> 1 +5 <u>6</u> 2 +5 <u>7</u> 3 +4 <u>7</u> 2 +9 <u>11</u> 8 +2 <u>10</u> 3 +4 <u>7</u>	7 +9 <u>16</u> 1 +4 <u>5</u> 4 +6 <u>10</u> 9 +5 <u>14</u> 4 +7 <u>11</u> 7 +4 <u>11</u> 1 +1 <u>2</u> 6 +5 <u>11</u> 5 +6 <u>11</u> 3 +8 <u>11</u> 8 +2 <u>10</u> 3 +9 <u>12</u> 5 +4 <u>9</u>	9 +7 <u>16</u> 9 +4 <u>13</u> 6 +9 <u>15</u> 4 +7 <u>11</u> 9 +8 <u>17</u> 4 +9 <u>13</u> 8 +3 <u>11</u> 5 +4 <u>9</u> 2 +1 <u>3</u> 4 +7 <u>11</u> 3 +2 <u>5</u> 8 +9 <u>17</u> 5 +4 <u>9</u>	9 +5 <u>14</u> 9 +5 <u>14</u> 4 +3 <u>7</u> 5 +8 <u>13</u> 7 +3 <u>10</u> 3 +6 <u>9</u> 7 +8 <u>15</u> 4 +4 <u>8</u> 3 +9 <u>12</u> 4 +2 <u>6</u> 4 +4 <u>8</u>
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so close!



Mental Math

$$367 + 421 = 461 + \underline{\hspace{2cm}}$$

Luke's Method

- What do we understand about Luke's understanding of numbers?
- What kind of thinking does he demonstrate in this example?

Their solution method...

$$367 + 421 = 461 + \underline{\hspace{2cm}}$$

You try it!

$$294 + 173 = \underline{\hspace{2cm}} + 283$$

In terms of Webb's Depths of Knowledge and Bloom's Taxonomy, where would these fall?

[illegible]

How do we help students develop fluency with their multiplication facts?



Number Talk

There are 28 girls and 35 boys on the playground. How many children are on the playground?

How can we use this problem to discuss multiplication?

Number Strings

What was
the purpose
of these
problems?

$$3 \times 5 = 15$$

$$2 \times 15 = 30$$

$$6 \times 15 = 90$$

$$12 \times 15 = 180$$

$$18 \times 15 = 270$$

Strategies for Multiplication

Break a factor into two or more addends

Round a factor and adjust

Factor a factor

Halving and doubling

Others?

Multiplication Strings

$$3 \times 4 = 12$$

$$3 \times 40 = 120$$

$$30 \times 40 = 1200$$

$$3 \times 0.4 = 1.2$$

$$3 \text{ cm} \times 4 \text{ cm} = 12 \text{ cm}^2$$

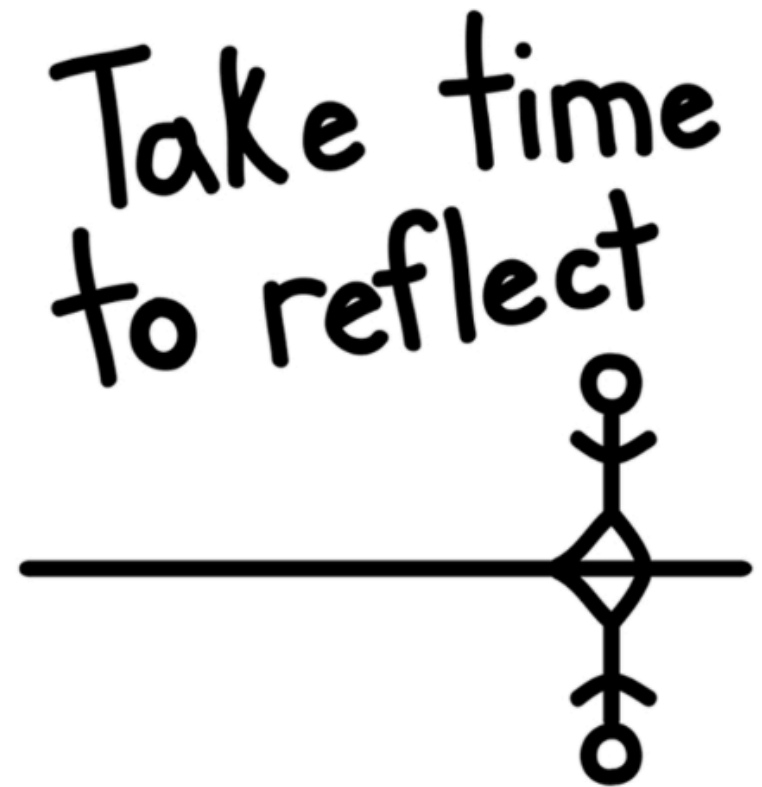
Rewrite each problem as
 $3 \text{ (units)} \times 4 \text{ (units)} = 12 \text{ (units)}$.

Fluency Revisited

Fluency demands more of students than does memorization of a single procedure. Fluency rests on a well-built mathematical foundation with three parts: (1) an understanding of the meaning of the operations and their relationships to each other...; (2) the knowledge of a large repertoire of number relationships, including the addition and multiplication "facts" as well as other relationships...; and (3) a thorough understanding of the base ten number system, how numbers are structured in this system, and how the place value system of numbers behaves in different operations.

From Jo Russell

What ideas about
fluency will you take
back to your
classroom?



Resources and References

Russell, Susan Jo. (May, 2000). Developing Computational Fluency with Whole Numbers in the Elementary Grades. In Ferrucci, Beverly J. and Heid, M. Kathleen (eds). Millenium Focus Issue: Perspectives on Principles and Standards. *The New England Math Journal*. Volume XXXII, Number 2. Keene, NH: Association of Teachers of Mathematics in New England. Pages 40-54. (http://investigations.terc.edu/library/bookpapers/comp_fluency.cfm)

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Boaler, Jo. *Fluency without fear*.

http://www.nctm.org/News-and-Calendar/Messages-from-the-President/Archive/Linda-M_-Gojak/Fluency_-_Simply-Fast-and-Accurate_-_I-Think-Not!/

Kling, Gina, and Jennifer M. Bay-Williams. "Assessing Basic Fact Fluency." *Teaching Children Mathematics* 20.8 (2014): 488-97. Web.

Thanks!
We welcome
your
feedback!

